Theoretical Vibrational Analysis of Automotive Exhaust Silencer

R.B. Patil* and V.P. Patekar**
Department of Mechanical Engineering,
*Jawaharlal Nehru College of Engineering, Aurangabad, (M.S.)
**Pad, Dr. Vitthalrao Vikhe Patil College of Engineering, Ahmednagar, (M.S.)

ABSTRACT: This paper in the first stage gives the introduction to the design analysis of an exhaust system. In the second part of the paper the mathematical model is developed and analysed for the detection of natural frequency of the exhaust system through mathematical calculations, these results are helpful so as to distinguish working frequency from natural frequency and avoid resonating condition.

I. INTRODUCTION

One of the objectives when designing a new automobile exhaust pipe is to lengthen it's durability period, which can be measured. in terms of its life span and mileage. The exhaust pipe is subjected to several stresses, most of which are due to vibration. Particular attention should be given to gas forces which will induce vibration. These vibrations will then induce a fatigue life to the system. It is therefore necessary to study the fatigue behavior of the exhaust pipe by analyzing the vibration modes and the response of vibrations by its sources [2].

II. NEED FOR ANALYSIS

The Automobile silencer under study belongs to a popular 2-Wheeler manufacturer in India with the rated HP of the engine up to @7.69HP. The exhaust gases coming out from engine are at very high speed and temperature. Silencer has to reduce noise, vibrations. While doing so it is subjected to thermal, vibration and fatigue failures which cause cracks. So it is necessary to analyze the vibrations which would further help to pursue future projects to minimize cracks, improving life and efficiency of silencer [3].

III. MATHEMATICAL MODELING

A mathematical model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed mathematical modeling. Mathematical models are used not only in the natural sciences (such as physics, biology, earth science, meteorology) and engineering disciplines (e.g. computer science, artificial intelligence), but also in the social sciences (such as economics, psychology, sociology and political science); physicists, engineers, statisticians, operations research analysts and economists use mathematical models most extensively. A model may help to explain a system and to study the effects of different components, and to make predictions about behavior.

Mathematical models can take many forms, including but not limited to dynamical systems, statistical models, differential equations, or game theoretic models. These and other types of models can overlap, with a given model involving a variety of abstract structures. In general, mathematical models may include logical models, as far as logic is taken as a part of mathematics. In many cases, the quality of a scientific field depends on how well the mathematical models developed on the theoretical side agree with results of repeatable experiments. Lack of agreement between theoretical mathematical models and experimental measurements often leads to important advances as better theories are developed.

Mathematical models are composed of variables, (abstractions of system parameters of interest), and operators (algebraic operators, functions, differential operators, etc.) which act on these variables.

A. Models Classification

Linear vs. nonlinear. If all the operators in a mathematical model exhibit linearity, the resulting mathematical model is defined as linear. A model is considered to be nonlinear otherwise. The definition of linearity and nonlinearity is dependent on context, and linear models may have nonlinear expressions in them. For example, in a statistical linear model, it is assumed that a relationship is linear in the parameters, but it may be nonlinear in the predictor variables. Similarly, a differential equation is said to be linear if it can be written with linear differential operators, but it can still have nonlinear expressions in it. In a mathematical programming model, if the objective functions and constraints are represented entirely by linear equations, then the model is regarded as a linear model.

If one or more of the objective functions or constraints are represented with a nonlinear equation, then the model is known as a nonlinear model. Nonlinearity, even in fairly simple systems, is often associated with phenomena such as chaos and irreversibility. Although there are exceptions, nonlinear systems and models tend to be more difficult to study than linear ones. A common approach to nonlinear problems is linearization, but this can be problematic if one is trying to study aspects such as irreversibility, which are strongly tied to nonlinearity.

Static vs. dynamic. A dynamic model accounts for time-dependent changes in the state of the system, while a static (or steady-state) model calculates the system in equilibrium, and thus is time-invariant. Dynamic models typically are represented by differential equations.

Explicit vs. implicit. If all of the input parameters of the overall model are known, and the output parameters can be calculated by a finite series of computations (known as linear programming, not to be confused with linearity as described above), the model is said to be explicit. But sometimes it is the output parameters which are known, and the corresponding inputs must be solved for by an iterative procedure, such as Newton's method (if the model is linear) or Broyden's method (if non-linear). For example, a jet engine's physical properties such as turbine and nozzle throat areas can be explicitly calculated given a design thermodynamic cycle (air and fuel flow rates, pressures, and temperatures) at a specific flight condition and power setting, but the engine's operating cycles at other flight conditions and power settings cannot be explicitly calculated from the constant physical properties.

Discrete vs. continuous. A discrete model treats objects as discrete, such as the particles in a molecular model or the states in a statistical model; while a continuous model represents the objects in a continuous manner, such as the velocity field of fluid in pipe flows, temperatures and stresses in a solid, and electric field that applies continuously over the entire model due to a point charge.

Deterministic vs. probabilistic (stochastic). A deterministic model is one in which every set of variable states is uniquely determined by parameters in the model and by sets of previous states of these variables. Therefore, deterministic models perform the same way for a given set of initial conditions. Conversely, in a stochastic model, randomness is present, and variable states are not described by unique values, but rather by probability distributions.

Deductive, inductive, or floating. A deductive model is a logical structure based on a theory. An inductive model arises from empirical findings and generalization from them. The floating model rests on neither theory nor observation, but is merely the invocation of expected structure. Application of mathematics in social sciences outside of economics has been criticized for unfounded models. Application of catastrophe theory in science has been characterized as a floating model [4].

IV. SPRING-MASS SYSTEMS

All systems possessing mass and elasticity are capable of free vibration, or vibration that takes place in the absence of external excitation. Of primary interest for such a system is its natural frequency of vibration. The basic vibration model of a simple oscillatory system consists of a mass, a mass less spring, and a damper. If damping in moderate amounts has little influence on the natural frequency, it may be neglected. The system can then be considered to be conservative. An undamped spring-mass system is the simplest free vibration system. It has one DOF [1].
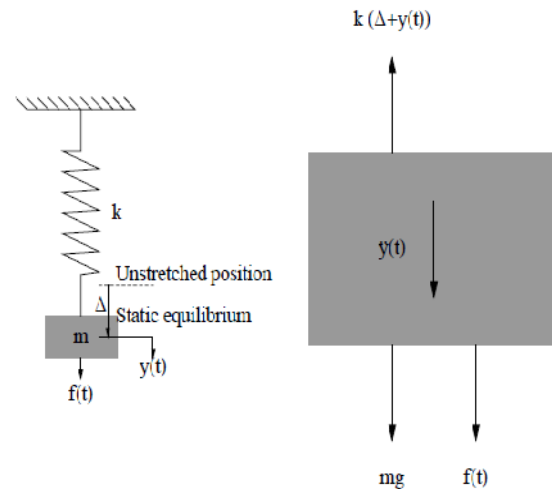

Fig.1. Spring Mass system.

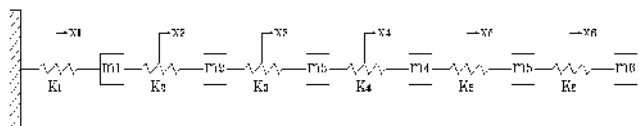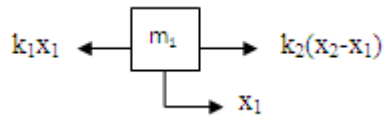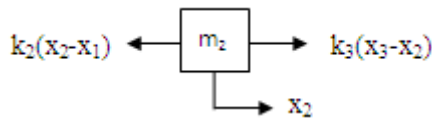The total system (silencer) is divided into the spring mass system as shown below,

Fig.2. Spring mass system for silencer.

The equation for each mass is written by considering the free body diagram of each mass element. The free body digs. For each mass is as follows [5],
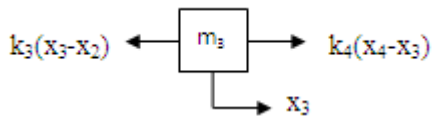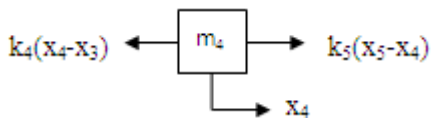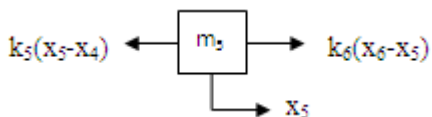
Therefore,

$$m_1\ddot{X}_1 + kx_1 - k(x_2 - x_1) = 0$$

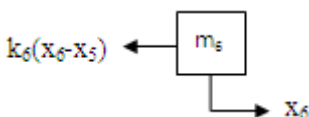
$$m_2\ddot{X}_2 + k(x_2 - x_1) - k(x_3 - x_2) = 0$$

$$m_3\ddot{X}_3 + k(x_3 - x_2) - k(x_4 - x_3) = 0$$

$$m_4\ddot{X}_4 + k(x_4 - x_3) - k(x_5 - x_4) = 0$$

$$m_5\ddot{X}_5 + k(x_5 - x_4) - k(x_6 - x_5) = 0$$

$$m_6\ddot{X}_6 + k(x_6 - x_5) = 0$$

The differential equation of spring mass system is as,

$$m\ddot{X} + kx = 0$$

The various matrices for system is as follows,

Stiffness matrix of the system is follows:-

K=

$$\begin{pmatrix} 2k & -k & 0 & 0 & 0 & 0 \\ -k & 2k & -k & 0 & 0 & 0 \\ 0 & -k & 2k & -k & 0 & 0 \\ 0 & 0 & -k & 2k & -k & 0 \\ 0 & 0 & 0 & -k & 2k & -k \\ 0 & 0 & 0 & 0 & -k & k \end{pmatrix}$$

$$(1/1e6) * \begin{bmatrix} 430 & -215 & 0 & 0 & 0 & 0 \\ -215 & 430 & -215 & 0 & 0 & 0 \\ 0 & -215 & 430 & -215 & 0 & 0 \\ 0 & 0 & -215 & 430 & -215 & 0 \\ 0 & 0 & 0 & -215 & 430 & -215 \\ 0 & 0 & 0 & 0 & -215 & 215 \end{bmatrix}$$

Mass matrix of the system is follows:-

M =

$$\begin{pmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_6 \end{pmatrix}$$

$$= \begin{bmatrix} 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.57 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix}$$

The differential equation of spring mass system is as,

$$[m][\ddot{X}] + [k][x] = 0$$

Multiplying equation by $[m]^{-1}$

$$[m]^{-1}[m][\ddot{x}] + [m]^{-1}[k][x] = 0$$

$$[I][\ddot{x}] + [D][x] = 0$$

Where, $[D] = [m]^{-1}[k]$

Also, $[x] = [X] \sin(\omega t)$
& $[\ddot{x}] = -\omega^2[x] = -\omega[X] \sin(\omega t)$

$$-\omega[I][x] + [D][x] = 0$$

$[D] - \lambda[I] = 0.$ ...(1)

$[D] = [m]^{-1}[k]$

Therefore the equation 1 becomes,

$$= \begin{pmatrix} 0.0007 & -0.0004 & 0 & 0 & 0 & 0 \\ -0.0003 & 0.0005 & -0.0003 & 0 & 0 & 0 \\ 0 & -0.0004 & 0.0008 & -0.0004 & 0 & 0 \\ 0 & 0 & -0.0004 & 0.0007 & -0.0004 & 0 \\ 0 & 0 & 0 & -0.0003 & 0.0006 & -0.0003 \\ 0 & 0 & 0 & 0 & -0.0022 & 0.0022 \end{pmatrix}$$

$$- \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{pmatrix}$$
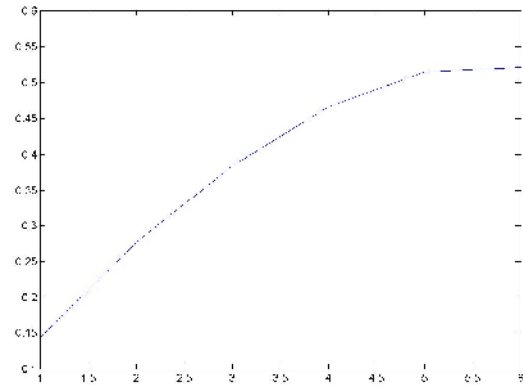
After solving the above equation, six different values of λ are obtained which gives six different natural frequency of the model. The different values of the λ are,

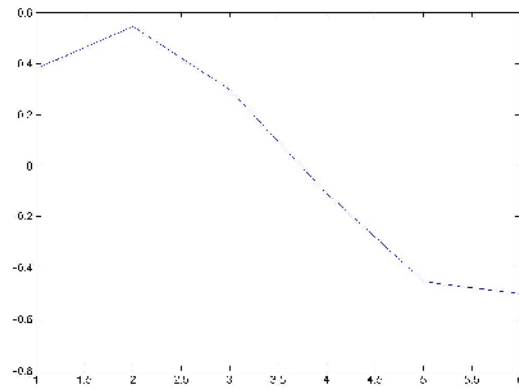$\lambda = 0e+004 \, [0.0398, 0.0802, 0.1079, 0.1733, 0.4936, 3.9470]$

Sample calculation: - (Frequency for sixth mode of vibration).
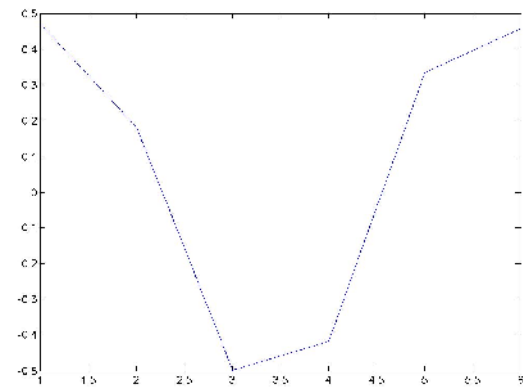$\lambda = \omega^2 = 0e+004 *(3.9470)$
$= 198.67$ Hz.
The different mode shape obtained is:-
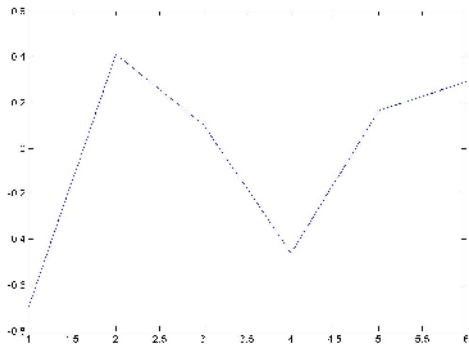

1st mode shape

2nd mode shape
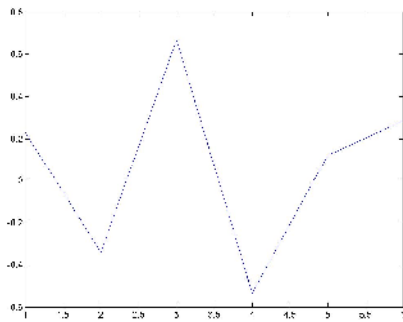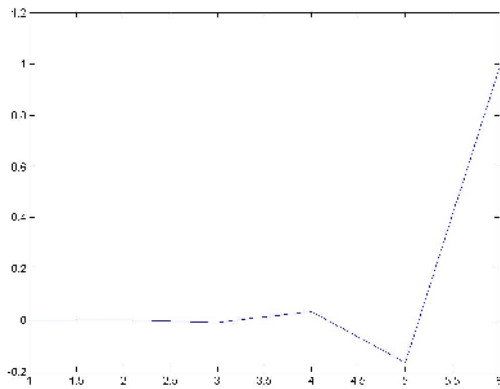

3rd mode shape

4th mode shape

Fig. 3. Different mode shape obtained through mathematical calculation.

5th mode shape

6th mode shape

Table 1: First six modal frequency.

Sr. No.	Freq. by Mathematical calculation.(in Hz.)
1	19.94
2	28.31
3	32.84
4	41.62
5	70.25
6	198.67

VI. CONCLUSION

The silencer natural frequencies have been calculated by mathematical calculations. By this method the natural frequencies obtained are useful while the design of silencer to avoid the resonance. Though the dynamic performance can be increased by increasing the thickness of different part. Furthermore is to add the support for partition, increase the support etc.

REFERENCES

[1] Amanda Frederick, Sarah Brady "Design project on vibration analysis on automobile muffler" April 23, 2003, EGR 315, *Mechanical Vibrations,* Dr.Amit Shukla, pp.1-15

[2] Shi Ying Wang Liang ,"The muffler suspension parts forepart fatigue damage problem study and improvement design, *2nd international Conference on Education Technology and Computer (ICETC),*vol.4, June 2010. pp. 1-4

[3] Wang Jie, Dong-peng Vue,"The Modal Analysis of Automotive Exhaust Muffler Based on PRO/E and ANSYS", *3rd International Conference on Advanced Computer Theory and Engineering (ICACTE),* 2010 pp.1-3.

[4] George Feng, Vadim Akishin, Bruce Huynh, "Flow considerations in Industrial Silencer Design. "Kinetics Noise Control, Inc., pp.1-8

[5] R.B. Patil and V.P. Patekar "Vibrational Analysis of Automotive Exhaust Silencer Based on FEM and FFT Analyzer" *International Journal on Emerging Technologies* **3**(2): 1-3(2012).

V. RESULT

The following table natural frequencies of vibration of silencer by mathematical calculations.